Research to Practice: Literature

- Skills in mathematics have been low as compared to other subject areas
- Low enrollment in more advanced mathematics classes
- Developmental delays
  - Flexibility in using numbers
    - Number sense
    - Mental number lines
  - Arithmetic combinations and equations of numbers
  - Decomposing numbers
- Lack conceptual, deep understanding of mathematics
- Lack procedural fluidity and strategy use
National Assessment of Educational Progress (2009)

• Assessment in 5 areas:
  - Number and Operations
  - Measurement
  - Geometry
  - Data Analysis
  - Algebra

• Scores in grade 4 have increased since 1990, but are not significantly different since 2007
  - 82% performing at the basic level
  - 39% performing at the proficient level
  - 6% performing at the advanced level
NMAP Recommendations

• Focused mathematics curriculum to meet the critical foundational needs for algebraic readiness
  - Fluency with whole number computations
  - Proficiency with Fractions
  - Aspects of Geometry and Measurement

• Core instruction is not enough!
  - Strategy instruction
  - Explicit and systematic instruction
  - Conceptual understanding
Progression of Mathematics Skills Towards Algebra Readiness, from Grades 5–8

FIFTH GRADE
- Proficiency with addition, subtraction, multiplication, and division of whole number algorithms
- Using fractions and decimals to represent, compare and order quantities, and solve problems
- Developing proficient use of whole number division standard algorithms to solve problems
- Using formulas to find area, volume, perimeter
- Understanding number lines, line graphs, and number plots
- Represent and compare whole numbers on the number line

SIXTH GRADE
- Fraction Equivalence
- Connecting ratios, rates, and proportions to multiplication and division and using these concepts and operations to solve problems involving proportional relationships
- Developing an understanding of rational numbers
- Using estimation to examine solutions of arithmetic problems
- Using expressions and equations to represent situations involving rational numbers
- Generalizations

SEVENTH GRADE
- Connecting and applying proportionality
- Fluency in examining rational numbers
- Using rational numbers and operations in a variety of contexts to solve problems
- Solving problems including determining angles, areas, perimeters, and volumes
- Using expressions and equations in a variety of contexts, including measurement, conversions, probability, and data analysis

EIGHTH GRADE
- Representing, applying, and analyzing proportionality
- Multiplicative thinking
- Using proportionality and linear equations to solve problems
- Problem solving including similarity, congruency, Pythagorean Theorem, and complex volumes
- Using expressions, equations, functions, and the real number system to represent and solve problems in a variety of contexts

ALGEBRA READINESS
- Mathematical Procedures and Skills
- Real numbers, fractions, decimals, use of algorithms
- Coordinate plane
- Problem solving
- Generalized Thinking
- Multiplicative/Proportional thinking
- Generalized notation
- Generalization of algorithms

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Mathematics Performance in Algebra Readiness

Percentages of Students who Scored “At or above Proficient” National Assessment of Educational Progress

National Assessment of Educational Progress (NAEP), 2009
Mathematics Performance in Algebra Readiness

- TAKS, State Data: Mathematics: Grade 5

<table>
<thead>
<tr>
<th></th>
<th>Number Tested</th>
<th>Percent Met Std Panel Rec.</th>
<th>Percent Commended Performance</th>
</tr>
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<tbody>
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<td>African-American:</td>
<td>43,352</td>
<td>75%</td>
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</tr>
<tr>
<td>Special Ed:</td>
<td>17,884</td>
<td>62%</td>
<td>17%</td>
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Mathematics Performance in Algebra Readiness

- TAKS, State Data: Mathematics: Grade 10

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<tr>
<td></td>
<td>Panel Rec.</td>
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<tr>
<td>African-American:</td>
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<tr>
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<td>Special Ed:</td>
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</tr>
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</table>
Procedures & Features: Tier II

- Identify an instructional sequence
  - Foundational
  - Multiple opportunities to practice within the lessons
    - Teach specific strategies
    - Build procedural knowledge
    - Increase student engagement
- Identify and teach prerequisite knowledge to build
  - Mastery
  - Fluency
- Quick pace
  - Use of time to stay on-task
  - Behavior management
- Error correction and scaffolds
Procedures & Features: Tier II

- Opportunities to make, show, write number concepts
- Enhance core curriculum through problem solving
- Regular, consistent intervention
  - 4-5 days per week
  - 20-30 minutes
- Progress monitoring
  - Daily
    - Independent Practice
    - 1-2 minutes
    - Reflect material taught
  - Weekly/Bi-weekly
    - Aim Checks
    - Generalization
Components of Explicit, Systematic Instruction

• All aspects of instruction include
  - Script
  - Time
  - Quick pace
  - Mix of choral and independent responses
  - Teacher talk decreases throughout lesson

• Preview/Cumulative Review (2-3 minutes)
  - Sets the tone for lesson
  - Purpose of lesson
  - Allows time to remediate/review skills
  - Choral and individual responses

• Modeled Practice (2-3 minutes)
  - Teaches the skill explicitly
  - Safe...students practice alongside or immediately following teacher directions
  - Quick pace
  - Small steps
Components of Explicit Instruction

- **Guided Practice (6-8 minutes)**
  - Similar to Modeled Practice
  - Increase individual time for practice
  - Provide error correction/scaffolds
  - Transaction from concrete to pictorial and/or pictorial to abstract
  - Teaches how to complete the daily check

- **Independent Practice (2-3 minutes)**
  - Fluency
  - Immediate feedback
  - Error correction

- **Total Time: 12-17 minutes**
Multiple Representations

- Concrete: Modeling/Guided Practice
  - Cubes
  - Counters
  - Base-ten/Place value materials (units, rods, flats)
  - Dot cubes
- Pictorial: Guided Practice/Independent Practice
  - Five frames
  - Ten frames
  - Hundreds charts
  - Number lines
- Abstract: Guided Practice/Independent Practice
  - Numbers
- Mats (can be used across all representations)
  - Part-part-whole
  - Fact family
  - Strategy mats
Student Verbalizations

- **Student Verbalizations** includes students’ thinking aloud about their problem solving approaches, mathematical understanding, or promoting mathematical discourse (Gersten et al., 2009)
  - Questions
  - Neighbor-share
  - Choral response
    - Wipe boards
    - Multiple choice answers
  - Identifying mistakes
Student Verbalizations

**Say:** What fraction is shown here? \( \frac{8}{12} \). I want to show that \( \frac{8}{12} \) is equivalent to \( \frac{2}{3} \). How would I do that?

Listen for students to say, “Combine 4 smaller parts to make larger parts.” As needed, give hints such as, “Remove division lines to make bigger parts.”

Circulate to monitor student progress. Randomly stop, draw attention to a completed cell, and ask: How did you get this answer? If you know this fact, what other fact do you know? Why?

Example: Draw attention to the cell containing 9 x 12 on the Practice 21 handout. Listen for a student to explain that 9 can be taken apart into 4 + 5, so the products from 4 x 12 and 5 x 12 were added to solve 9 x 12 because the 9s facts can be found by taking apart the 9s facts into 4s facts plus the 5s facts. Listen for students to explain that if 9 x 12 is known, 12 x 9 is also known because of the commutative property of multiplication and watch for students to complete the row and column.
**Visual Representations**

- **Using Visual Representations** used during instruction include those used by the teacher to model problem solving as well as student use of manipulatives (Gersten et al., 2009).
2. A guitar has 6 strings. Adam has 8 guitars. Draw a number line to show how many guitar strings there are in all.

There are 48 strings in all.
3 Tier Mathematics Model

- Free to Texas Educators at http://3tiermathmodel.org/
- Username: Texas Teacher
- Password: mathematics
Content and Skills: Grades K-2nd

- **Word Problem Solving:**
  - Strategy to solve all types
  - Different types of problems
  - Extraneous information
  - Multiple steps, contextualized

- **Number Knowledge and Relationships**
  - Counting: Rote, Rational, Counting Up/Back, Skip (2, 5, 10)
  - Number Recognition & Writing: 0-20 (kinder) 0 - 99 (1st); 0 - 999 (2nd grade)

- **Number Relationships of greater than/less than/equal to**
  - Relationships of one and two more than/less than
  - Anchoring Numbers to 5 & 10 frames
  - Part-part-whole Relationships (e.g., ways to represent numbers)

- **Numeric Sequencing**
  - Number line, mental number line
  - Math flexibility
  - Ordering numbers
Grades K-2\textsuperscript{nd} (cont.)

- Base 10 & Place Value
  - Making and counting:
    - Groups of tens and ones (1st grade)
    - Groups of hundreds, tens, and ones (2nd grade)
    - Using base-ten language (3 hundreds, 0 tens, 6 ones) and standard language (306) to describe place value
  - Reading and writing numbers to represent base ten models
  - Naming the place value held by digits in numbers

- Addition & Subtraction Combinations
  - Identity Element and Properties
  - Fact Families
  - Counting & Decomposition Strategies
  - Addition: count on, [+ 0, + 1, + 2], doubles, doubles +1, make 10 + more
  - Subtraction: count down [-0, -1, -2, -3], count on
Intervention Modules

**Grade 3**
- Place Value Concepts
- Addition & Subtraction of Whole Numbers
- Multiplication & Division Concepts
- Fraction Concepts

**Grade 4**
- Multiplication & Division Strategies
- Multiplication & Division of Whole Numbers
- Modeling, Comparing, & Ordering Fractions
- Fraction & Decimal Relationships
The students is expected to:
• Use place value to read, write (in symbols and words) and describe the value of whole numbers through 999,999. (3.1 A, supporting)
• Use place value to compare and order whole numbers through 9,999. (3.1 B, supporting)
• Identify and extend whole-number patterns. (3.6 A; supporting)
• Use data to describe events as more likely than, less likely than, or equally likely as. (3.13 C, supporting)
• Applies Grade 3 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (3.14 A; B; C; D)
• Can relate informal language to mathematical language and symbols. (3.15 B)

Module A: Place Value Concepts

Module B: Addition & Subtraction of Whole Numbers

Module C: Multiplication and Division Concepts

Module D: Fraction Concepts
### Module A: Multiplication & Division Strategies

The students is expected to:

- Use multiplication and division to solve problems without technology. (4.4 D, E, readiness)
- Model factors and products using arrays and area models. (4.4 A, supporting; 4.4 B, supporting)
- Represent multiplication and division situations in picture, word and number form. (4.4 B, supporting)
- Use multiplication and division to solve problems (4.4 D, E, readiness)
- Use patterns and relationships to develop strategies to remember basic multiplication and division facts (4.4 A, supporting)
- Represent multiplication and division situations in picture, word, and number form. (4.4 B, supporting)
- Recall and apply multiplication facts through 12 x 12. (4.4 C, supporting)
- Apply Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
- Justify why an answer is reasonable and explain the solution process. (4.16 B)

### Module B: Multiplication & Division of Whole Numbers

The students is expected to:

- Use multiplication and division to solve problems (no more than 2 digits times 2 digits or one digit divisor and three-digit dividends without technology). (4.4 D; E, readiness)
- Model factors and products using arrays and area models (4.4 A, supporting)
- Represent multiplication and division situations in picture, word and number form. (4.4 B, supporting)
- Use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems (4.5 B, supporting)
- Use patterns to multiply by 10 and 100. (4.6 B, supporting)
- Apply Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
- Justify why an answer is reasonable and explain the solution process. (4.16 B)

### Module C: Modeling, Comparing & Ordering Fractions

The students is expected to:

- Model fraction quantities greater than one using concrete objects and pictorial models. (4.2 B, supporting)
- Use concrete objects and pictorial models to generate equivalent fractions. (4.2 A, supporting)
- To locate and name points on a number line using whole numbers and fractions such as halves and fourths. (4.10 A, readiness)
- Compare and order fractions using concrete objects and pictorial models. (4.2 C, supporting)
- Apply Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
- Justify why an answer is reasonable and explain the solution process. (4.16 B)

### Module D: Fraction & Decimal Relationships

The students is expected to:

- Relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models. (4.2 D, readiness)
- Locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths. (4.10 A, readiness)
- Relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models. (4.2 D, readiness)
- Use place value to read, write, compare and order decimals involving tens and hundredths, including money, using concrete objects and pictorial models. (4.1 B, readiness; 4.2 C, supporting)
- Apply Grade 4 math to solve problems connected to everyday experiences in and outside of school. The student is expected to understand the problem, make a plan, carry out the plan and evaluate the solution for reasonableness. (4.14 A; B; C; D)
- Can relate informal language to mathematical language and symbols. (4.15 B)
- Make generalizations from patterns or sets of examples and nonexamples (4.16 A)
- Justify why an answer is reasonable and explain the solution process. (4.16 B)
Intervention 6th – 9th Grade

• Project Share
  - MSTAR: Middle School Intervention
    • Facts & Patterns in Multiplication and Division
    • Proportionality
    • Ratios and Rates
    • Equivalent Fractions
  - H.S. Algebra Intervention Modules
    • Variables
    • Expressions, Equations, & Equivalence
Students Who Struggle...

NMAP Progressions

Table 2: Benchmarks for the Critical Foundations

<table>
<thead>
<tr>
<th>Fluency With Whole Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.</td>
</tr>
<tr>
<td>2) By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fluency With Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.</td>
</tr>
<tr>
<td>2) By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percent, and with the addition and subtraction of fractions and decimals.</td>
</tr>
<tr>
<td>3) By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.</td>
</tr>
<tr>
<td>4) By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.</td>
</tr>
<tr>
<td>5) By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.</td>
</tr>
<tr>
<td>6) By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.</td>
</tr>
</tbody>
</table>

Geometry and Measurement

1) By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
2) By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
3) By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

The Major Topics of School Algebra

Symbols and Expressions
- Polynomial expressions
- Rational expressions
- Arithmetic and finite geometric series

Linear Equations
- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations
- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions
- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions, absolute value, rational functions, step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data

Algebra of Polynomials
- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal’s Triangle)
- Mathematical induction and the binomial theorem

Combinatorics and Finite Probability
- Combinations and permutations, as applications of the binomial theorem and Pascal’s Triangle

~ Taken from National Mathematics Advisory Panel, 2008, p. 16, 20